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s question paper contains 8 printed pages.]

Your Roll No...............

No. of Question Paper : 1608 A

que Paper Code : 42357602

e of the Paper : DSE – Probability and Statistics

e of the Course : B.Sc. Mathematical Sciences

ester : VI

tion : 3 Hours Maximum Marks : 75

ructions for Candidates

Write your Roll No. on the top immediately on receipt of this question paper.

Attempt all the six questions.

Each question has four parts. Attempt any two parts from each question.

Each part in Question 1, 3, 5 carries 6 marks.

Each part in Question 2, 4, 6 carries 6.5 marks.

Use of scientific calculator is allowed.

(a) A secretary types three letters and three corresponding envelopes. In a hurry, he places at random one letter in each envelope. What is the probability that at least one letter is in correct envelope?

P.T.O.

(b) Let $\{C_n\}$ be a nondecreasing sequence of events. Show that

$$\lim_{n\to\infty} P(C_n) = P\left(\lim_{n\to\infty} C_n\right) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

(c) Let $p_X(x)$ be the pmf of a random variable X. Find and sketch the cdf $F_X(x)$ of X, where

$$p_X(x) = \begin{cases} \dfrac{x}{15} & x = 1, 2, 3, 4, 5 \\ 0 & elsewhere \end{cases}$$

Also find

(i) $P(X = 1 \text{ or } 2)$

(ii) $P\left(\frac{1}{2} < X < \frac{5}{2}\right)$

(iii) $P(1 \le X \le 4)$

(d) Define cumulative distribution function. Let X be a random variable with cumulative distribution function F(x). Show that

$$\lim_{x \downarrow x_0} F(x) = F(x_0)$$

for all $x_0 \in \mathbb{R}$.

2. (a) A bowl contains 10 chips, of which 8 are marked \$2 each and 2 are marked \$5 each. A person chooses three chips at random without replacement from this bowl. If person is to receive the sum of the resulting amounts. Find his expectation.

(b) Find the moment generating function of Normal Distribution. Also, find its mean and variance using moment generating function.

(c) Let a random variable of continuous type have a pdf f(x) whose graph is symmetric with respect to the line x = c. If the mean value of X exists. Show that E(X) = c.

(d) If the probability is 0.75 that a person will believe a rumor about a certain actor. Find the probability that

(i) the fifth person to hear the rumor will be the second to believe it.

(ii) the sixth person to hear the rumor will be the fourth to believe it.

3. (a) Find the joint probability density of the two random variables X and Y whose joint distribution function is given by :

P.T.O.

$$F_{X,Y}(x,y)=\begin{cases}\left(1-e^{-x^2}\right)\left(1-e^{-y^2}\right); & x>0,\ y>0\\ 0 & ;\ \text{otherwise}\end{cases}$$

Then use the joint pdf to obtain $P[1 < X \le 2, 1 < Y \le 2]$.

(b) Let X and Y have the joint probability mass function :

$$p(m,n)=\begin{cases}\dfrac{1}{2^{(m+1)}}; & m \ge n\\ 0 & ;\ m < n\end{cases}$$

for m, n = 1, 2,...... Verify that it satisfies the properties of a joint pmf. Compute the marginal probability mass functions.

(c) Consider the experiment of tossing two tetrahedra with sides numbered 1 to 4. Let X denotes the smaller of the two downturned numbers and Y be the larger.

(i) Find the joint mass function of X and Y.

(ii) Find $P[X \ge 2, Y \ge 2]$.

(iii) Find conditional pmf of X given Y.

(d) Suppose that X and Y are jointly continuous random variables with joint density :

$$f_{X,Y(x,y)} = \begin{cases} c\,x^2y : & 0 < x < y < 2 \\ 0 : & \text{otherwise} \end{cases}$$

(i) Find the value of c ?

(ii) What is the probability that $X < 2Y$?

(iii) What are the marginal densities f_X and f_Y ?

4. (a) Let X and Y be two random variables with joint pdf :

$$f(x,y) = \begin{cases} 5\,xy : & 0 < x,\ y < 1 \\ 0 : & \text{otherwise} \end{cases}$$

(i) Find joint moment generating function of X and Y.

(ii) Using joint mgf, compute E(XY) and E(X).

(iii) Compute E(2X – 4XY).

(b) If the joint probability density of X and Y is given by :

$$f(x, y) = \begin{cases} 24y(1-x-y) & x > 0, y > 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute conditional mean and variance of Y given $X = x$, $x > 0$.

(c) Let X and Y be discrete random variables. Then prove the following :

(i) $E[E(Y \mid X)] = E(Y)$

(ii) $\text{var}(Y) = E[\text{var}(Y \mid X)] + \text{var}(E(Y \mid X))$.

(d) If two cards are randomly drawn (without replacement) from an ordinary deck of 52 playing cards, X is the number of spades obtained in the first draw, and Y is the total number of spades obtained in both draws, find

(i) the joint cumulative distribution function of X and Y;

(ii) conditional cdf of X given $Y = y$.

5. (a) If X is a random variable with mean μ and variance σ^2, then prove that for any k > 0

$$P\left[|X-\mu| < k\sigma\right] \geq 1-\frac{1}{k^2}.$$

(b) Use the Central limit theorem to prove that if X is a random variable having binomial distribution with parameters n and θ, then

$$\frac{X-n\theta}{\sqrt{n\theta(1-\theta)}} \sim N(0,1) \text{ as } n\to\infty.$$

(c) Given the joint density

$$f(x,y)=\begin{cases}2 & \text{for } 0<y<x<1\\ 0 & \text{elsewhere}\end{cases}$$

Find the regression equation of Y on X.

(d) Using method of least squares fit a straight line for the following data

X	1	2	3	4	6	8
Y	2.4	3	3.6	4	5	6

6. (a) Let X and Y have joint probability mass function described as follows

P.T.O.

(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)
p(x, y)	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

Find the coefficient of correlation of X and Y.

(b) Find $P\left(0 < X < \frac{1}{3}, 0 < Y < \frac{1}{3}\right)$, if the random variable X and Y have joint probability density function

$$f(x,y) = \begin{cases} 4x(1-y), & 0 < x < 1, \ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(c) If the regression of Y on X is linear, then show that

$$E[Y|X] = \mu_2 + \frac{\rho\sigma_2}{\sigma_1}(X - \mu_1).$$

(d) Let X_i, i = 1 to 5 be independent random variables, each being uniformly distributed over (0,1). Use the Markov's inequality to get bound on

$$P[X_1 + X_2 + X_3 + X_4 + X_5 \geq 8].$$

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